STATISTICS II

Bachelor's degrees in Economics, Finance and Management

2nd year/2nd Semester
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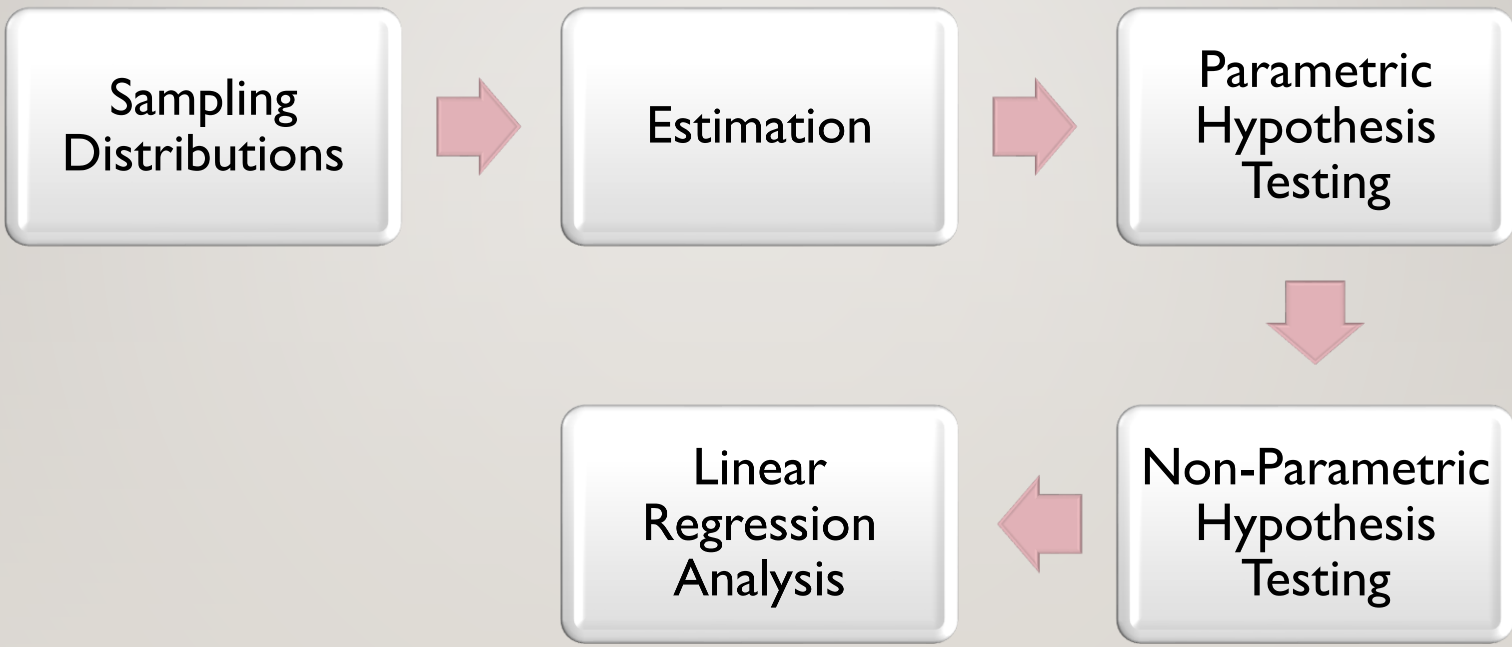
https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM


Sampling Distributions
Estimation
Parametric Hypothesis Testing
Non-Parametric Hypothesis Testing
Linear Regression Analysis

HOMEWORK OF LECTURE 13: QUESTIONS AND SOLUTIONS

EXERCISE 9.11

9.11 A manufacturer of detergent claims that the contents of boxes sold weigh on average at least 16 ounces. The distribution of weight is known to be normal, with a standard deviation of 0.4 ounce. A random sample of 16 boxes yielded a sample mean weight of 15.84 ounces. Test at the 10% significance level the null hypothesis that the population mean weight is at least 16 ounces.

Newbold et al (2013)

EXERCISE 9.11: SOLUTION

Answer:

Left-Tailed Test

Step 1: State the hypotheses

$$H_0 : \mu \geq 16 \quad \text{(the mean weight is at least 16 oz)}$$

$$H_1 : \mu < 16 \quad \text{(the mean weight is less than 16 oz)}$$

This is a **left-tailed** test.

Step 2: Given information

$$\sigma = 0.4, \quad n = 16, \quad \bar{x} = 15.84, \quad \alpha = 0.10$$

EXERCISE 9.11: SOLUTION

Answer:

Step 3: Test Statistic

Since the population standard deviation is known and the population is normal, we use a

Z-test: $z_{1-\alpha/2}$

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{15.84 - 16}{0.4/\sqrt{16}}$$

$$Z = \frac{-0.16}{0.1} = -1.6$$

Step 4: Critical Value / p-value

Left-Tailed Test: RR = $]-\infty; z_\alpha]$

RR = $]-\infty; -1.282]$

- Left-tailed test, α = 0.10 → $z_{0.10} \approx -1.282$

Standard Normal Distribution Table

P-value = P(Z < -1.6) ~ 0.055

EXERCISE 9.11: SOLUTION

Answer:

Step 5: Decision Rule

Compare the test statistic to the critical value:

$$Z = -1.6 \quad \text{is less than} \quad -1.28$$

- Calculated z = -1.6 < -1.282 → **reject** H_0
- p-value ≈ 0.055 < 0.10 → **reject** H_0

$-1.6 \in RR =]-\infty; z_\alpha] =]-\infty; -1.282]$

Conclusion

There is **sufficient evidence at the 10% significance level** to conclude that the mean weight of the detergent boxes is **less than 16 ounces**. The manufacturer's claim is **not supported**.

Note:
In the following slides, we will examine **both the right-tailed and two-tailed tests** to compare the results.

EXERCISE 9.11:TYPES OF HYPOTHESIS TESTS FOR THE MEAN (σ^2 KNOWN)

✅ Answer:

1 Left-tailed

- $H_0 : \mu \geq 16$
- $H_1 : \mu < 16$

> Test whether the mean weight is less than 16 ounces.

2 Right-tailed

- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$

> Test whether the mean weight is greater than 16 ounces.

3 Two-tailed

- $H_0 : \mu = 16$
- $H_1 : \mu \neq 16$

> Test whether the mean weight differs from 16 ounces.

EXERCISE 9.11: RIGHT-TAILED TEST

Answer:

Problem Setup:

- Population standard deviation: σ = 0.4
- Sample size: n = 16
- Sample mean: $\bar{x} = 15.84$
- Hypotheses:

Right-Tailed Test

$$H_0 : \mu \leq 16 \quad , \quad H_1 : \mu > 16$$

- Significance level: α = 0.10

Test statistic (z):

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{15.84 - 16}{0.4/\sqrt{16}} = \frac{-0.16}{0.1} = -1.6$$

Critical value: $z_{0.9} = 1.282$

Decision rule: Reje

Right-Tailed Test: RR = $[z_{1-\alpha}; +\infty[$ 	RR = $[1.282 +\infty[$

Conclusion:

P-value = P(Z > -1.6) = 1- P(Z < -1.6) ~ 1 – 0.0548 = 0.9452

- **Critical value:** z = -1.6 < 1.282 → do not reject H_0
- **p-value:** 0.945 > 0.10 → do not reject H_0

$-1.6 \notin$ RR = $[1.282, +\infty[$

- **Interpretation:** Not enough evidence to conclude the mean weight is **greater than 16** ounces.

EXERCISE 9.11: TWO-TAILED TEST

✅ Answer:

Problem Setup:

- Same data as above
- Hypotheses:

Two-Tailed Test

$$H_0 : \mu = 16 \quad , \quad H_1 : \mu \neq 16$$

- Significance level: α = 0.10

Test statistic (z):

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -1.6$$

Critical values:

- Two-tailed: $z_{0.05} = \pm 1.645$

Two-Tailed Test : RR =] -∞; $-z_{1-\alpha/2}$]U[$z_{1-\alpha/2}$; +∞[

RR =] -∞; -1.645]U[1.645; +∞[

Decision

P-value = 2×P(Z < -1.6) ~ 2× 0.0548 = 0.1096

Conclusion:

- **Critical value:** |z| = 1.6 < 1.645 → do not reject H_0
- **p-value:** 0.110 > 0.10 → do not reject H_0

⟺ -1.6 ∉ RR =] -∞; -1.645]U[1.645; +∞[

- **Interpretation:** Not enough evidence to conclude the mean weight **differs from 16** ounces.

EXERCISE 9.12

9.12 A company that receives shipments of batteries tests a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with a standard deviation of 3 hours. For one particular shipment the mean lifetime for a sample of nine batteries was 48.2 hours. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.

Newbold et al (2013)

EXERCISE 9.12: SOLUTION

Answer:

Left-Tailed Test

Step 1 — Hypotheses

$$H_0 : \mu \geq 50 \quad \text{vs} \quad H_1 : \mu < 50$$

(Left-tailed test.)

Step 2 — Data

$$\sigma = 3, \quad n = 9, \quad \bar{x} = 48.2, \quad \alpha = 0.10$$

Since σ is known and the population is normal, use the Z-test.

EXERCISE 9.12: SOLUTION

✅ Answer:

Step 3 — Test statistic

Standard error:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{9}} = 1$$

Test statistic:

$$Z = \frac{\bar{x} - \mu_0}{SE} = \frac{48.2 - 50}{1} = -1.8$$

Step 4 — Critical value / p-value

Critical value for left-tailed test at $\alpha = 0.10$:

$$Z_{critical} \approx -1.28$$

RR =] -∞; -1.28]

Comparison: $Z = -1.8 < -1.28$ → falls in the rejection region.

p-value: P-value = P(Z < -1.8) ~ 0.0359

EXERCISE 9.12: SOLUTION

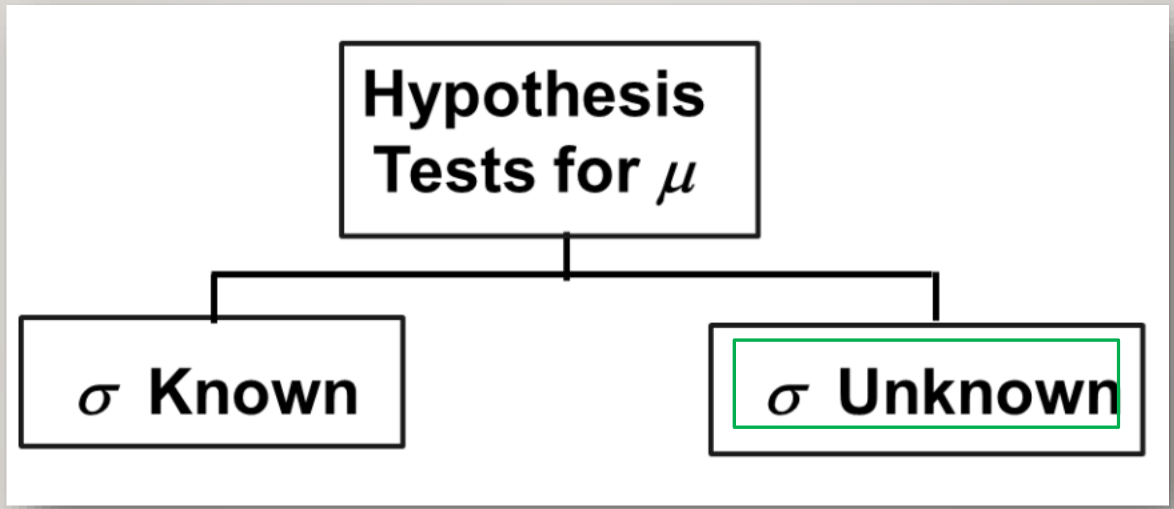
Answer:

Step 5 — Decision and conclusion

- Because $Z = -1.8$ is less than -1.28 (and $p\text{-value} \approx 0.036 < 0.10$), we **reject** H_0 at the 10% significance level.
- **Conclusion:** There is sufficient evidence at the 10% level to conclude the population mean lifetime is **less than 50 hours**. The shipment does not meet the company's requirement.

LECTURE 14: TESTS OF THE MEAN OF A NORMAL DISTRIBUTION (σ^2 UNKNOWN)

HYPOTHESIS TESTS FOR THE MEAN



Newbold et al (2013)

TESTS OF THE MEAN OF A NORMAL DISTRIBUTION (σ^2 UNKNOWN): EXAMPLE

- Convert sample result $(\bar{x})$ to a *t* test statistic

Note: When performing a hypothesis test for the **mean with unknown variance**, it is assumed that the **data come from a normally distributed population** (especially when the sample size is small). In this case, the **t-distribution** is used.

Hypothesis Tests for μ

σ Known

σ Unknown

Consider the test

Right-Tailed Test

$H_0: \mu \le \mu_0$

$H_1: \mu > \mu_0$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{1-\alpha;\, n-1}$$

Newbold et al (2013)

TESTS OF THE MEAN OF A NORMAL DISTRIBUTION (σ^2 UNKNOWN): EXAMPLE

- For a two-tailed test:

Consider the test

Two-Tailed Test

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

(Assume the population is normal, and the population variance is unknown)

The decision rule is:

Reject H_0 if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} < -t_{1-\alpha/2;\, n-1}$ or if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{1-\alpha/2;\, n-1}$

Newbold et al (2013)

TWO-TAILED TEST (σ^2 UNKNOWN): EXAMPLE 1

The average cost of a hotel room in Chicago is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x} = \$172.50$ and $s = \$15.40$. Test at the $\alpha = 0.05$ level.

Two-Tailed Test

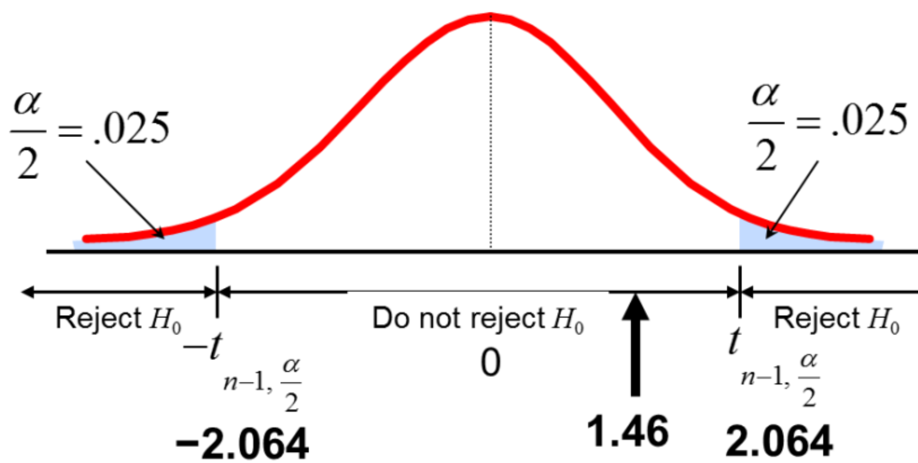
$$H_0 : \mu = 168$$

$$H_1 : \mu \neq 168$$

Newbold et al (2013)

TWO-TAILED TEST (σ^2 UNKNOWN): EXAMPLE SOLUTION

$$H_0 : \mu = 168$$
$$H_1 : \mu \neq 168$$



- $\alpha = 0.05$
- $n = 25$
- σ is unknown, so use a t statistic →

$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

- **Critical Value:**

$$t_{24, .025} = \pm 2.064$$

Decision Using the Rejection Region (RR)

Do not reject H_0 : not sufficient evidence that true mean cost is different than \$168

Two-Tailed Test : RR =] -∞; $-t_{1-\alpha/2}$]U[$t_{-\alpha/2}$; +∞[

RR =] -∞; -2.064]U[2.064; +∞[

Note (Two-Tailed Test): The value of the test statistic (1.46) does **not** fall in the rejection region. In this case, we **fail to reject** H_0.

Newbold et al (2013)

EXERCISE 9.25

9.25 A statistics instructor is interested in the ability of students to assess the difficulty of a test they have taken. This test was taken by a large group of students, and the average score was 78.5. A random sample of eight students was asked to predict this average score. Their predictions were as follows:

72 83 78 65 69 77 81 71

Assuming a normal distribution, test the null hypothesis that the population mean prediction would be 78.5. Use a two-sided alternative and a 10% significance level.

Newbold et al (2013)

EXERCISE 9.25: SOLUTION

One-Sample t-Test for the Mean (Unknown Variance)

✅ Answer:

Step 1 – Problem Setup

- Sample size: $n = 8$
- Sample mean: $\bar{x} = 74.5$
- Sample standard deviation: $s \approx 6.23$
- Null hypothesis: $H_0 : \mu = 78.5$
- Alternative hypothesis: $H_1 : \mu \neq 78.5$
- Significance level: $\alpha = 0.10$
- Test: **two-tailed t-test** (σ unknown, small sample)

Two-Tailed Test

Step 2 – Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{74.5 - 78.5}{6.23/\sqrt{8}} \approx -1.82$$

Degrees of freedom: $df = n - 1 = 7$

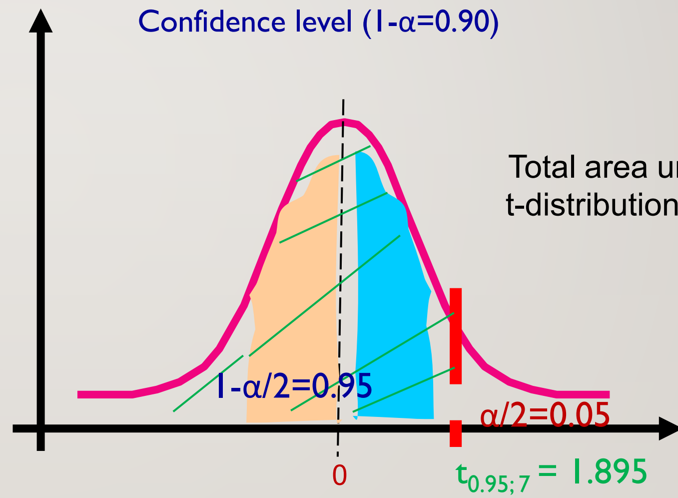
CRITICAL VALUE $t_{1-\alpha/2;\,n-1}$: CALCULATION

Two-Tailed Test → RR =] -∞; $-t_{1-\alpha/2}$]U[$t_{1-\alpha/2}$; +∞[?

n \ ε	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.214
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.894
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261
			1.296	1.671	2.000			
			1.294	1.667	1.994			
			1.292	1.664	1.990			
			1.291	1.662	1.987			
			1.290	1.660	1.984			
			1.289	1.658	1.980			
			1.282	1.645	1.960			

Significance level (α=0.10)

Confidence level (1-α=0.90)



Total area under the t-distribution pdf = 1.

RR =] -∞; -1.895]U[1.895; +∞[

Note:
The Student's t table reports right-tail probabilities: P(T > t).

α = 0.10

1 - α/2 = 0.95

$t_{1-\frac{\alpha}{2};n-1}$ = $t_{0.95;7}$ = 1.895

P-VALUE FOR A STUDENT´S T-STATISTIC: CALCULATION

Two-Tailed Test ➔ P-value = $2 \times P(T \geq |t_0|)$

The value of the test statistic is $t_0 = -1.82$

n \ ε	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
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16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646
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21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527
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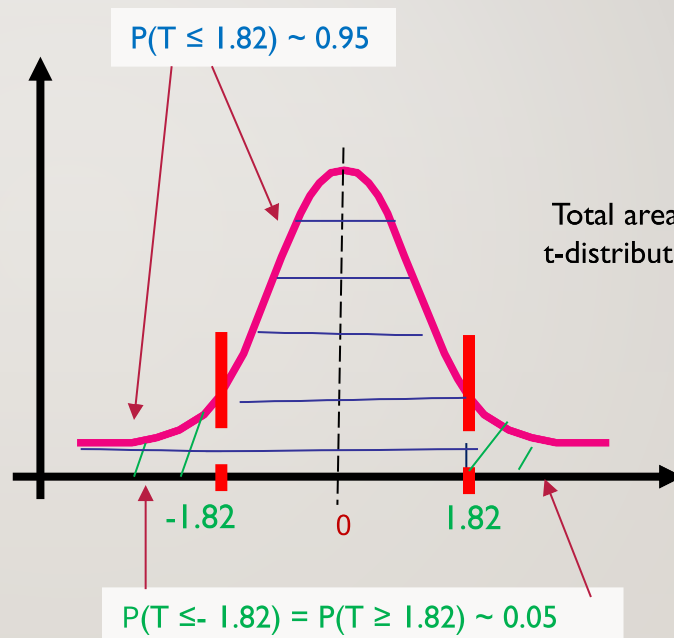
				1.703	2.052	2.473	2.771	3.421
				1.701	2.048	2.467	2.763	3.408
				1.699	2.045	2.462	2.756	3.396
				1.697	2.042	2.457	2.750	3.385
				1.684	2.021	2.423	2.704	3.307
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261
					00	2.390	2.660	3.232
					94	2.381	2.648	3.211
					90	2.374	2.639	3.195
					87	2.368	2.632	3.183
					84	2.364	2.626	3.174
					80	2.358	2.617	3.160
					60	2.326	2.576	3.090

Note:
The Student's t table reports right-tail probabilities: $P(T > t)$.

P-value = $2 \times P(T \geq |t_0|)$ ⇔
P-value = $2 \times P(T \geq 1.82)$ ⇔
P-value ~ $2 \times P(T \geq 1.895)$ ⇔
P-value ~ $2 \times 0.05 = 0.1$



Total area under the t-distribution pdf = 1.

P-value = sum of the two green areas = 0.05 + 0.05 = 0.1

EXERCISE 9.25: SOLUTION

Answer:

Note:
The rejection region and the p-value were calculated in the two previous slides, respectively.

Step 3 – Critical Value

RR =] -∞; -1.895]U[1.895; +∞[

- Two-tailed test, α = 0.10 → $t_{0.05,7} \approx \pm 1.895$

Decision rule: Reject H_0 if $|t| > 1.895$

Step 4 – p-value

- Using t-distribution with 7 df:

P-value = 0.1
Approximate value (from the Student's t-distribution table)

$$p\text{-value} = 2 \cdot P(T > |t|) \approx 2 \cdot P(T > 1.82) \approx 0.11$$

Exact value

Step 5 – Conclusion

- Calculated t = -1.82 → $|t| < 1.895$
- p-value ≈ 0.11 > 0.10

Decision: Do not reject H_0

Decision rule: Reject H_0 if the test statistic t is in the rejection region (RR) or if p-value < α.

Note:
In the following slides, we will examine **both the left-tailed and right-tailed tests** to compare the results.

Interpretation: There is **not enough evidence** at the 10% significance level to conclude that the mean predicted score differs from 78.5.

EXERCISE 9.25: LEFT-TAILED TEST

One-Sample t-Test for the Mean (Unknown Variance)

Answer:

Step 1 – Problem Setup

Left-Tailed Test

- Sample size: $n = 8$
- Sample mean: $\bar{x} = 74.5$
- Sample standard deviation: $s \approx 6.23$
- Degrees of freedom: $df = 7$
- Test statistic: $t \approx -1.82$

Step 2 – Left-tailed test

- Null hypothesis: $H_0 : \mu \geq 78.5$
- Alternative hypothesis: $H_1 : \mu < 78.5$
- Significance level: $\alpha = 0.10$

Critical value: $t_{0.10,7} \approx -1.415$ RR =] -∞; -1.415]

Decision rule: Reject H_0 if $t < -1.415$

p-value: P(T < -1.82) = 0.058 Exact value

Note:
The value of the test statistic is the same for both two-tailed and one-tailed tests: $t = -1.82$.

Note:
The rejection region and the p-value will be computed in the next two slides, respectively.

Conclusion:

- P-value ~ 0.058 < 0.1 → reject H_0
- t = -1.82 < -1.415 → **reject** H_0
- There is **enough evidence** at the 10% significance level to conclude that the mean predicted score is **less than 78.5**.

CRITICAL VALUE $t_{\alpha;\, n-1}$: CALCULATION

Left-Tailed Test → RR =] -∞; t_α] ?

ε n	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
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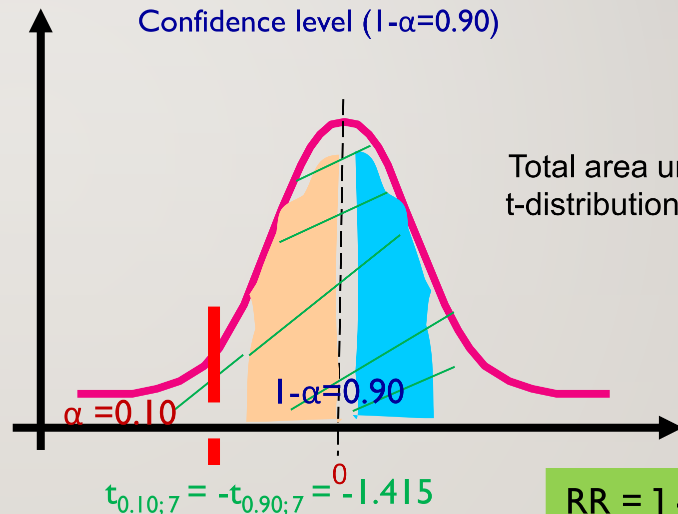
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232
				1.667	1.994	2.381	2.648	3.211
				1.664	1.990	2.374	2.639	3.195
				1.662	1.987	2.368	2.632	3.183
				1.660	1.984	2.364	2.626	3.174
				1.658	1.980	2.358	2.617	3.160
				1.645	1.960	2.326	2.576	3.090

Note:
The Student’s t table reports right-tail probabilities: P(T > t).

Significance level (α=0.10)

Confidence level (1-α=0.90)



RR =] -∞; -1.415]

α = 0.10

1 - α = 0.90

$t_{\alpha;n-1}$ = $-t_{1-\alpha;n-1}$ = $-t_{0.90;7}$ = -1.415

P-VALUE FOR A STUDENT´S T-STATISTIC: CALCULATION

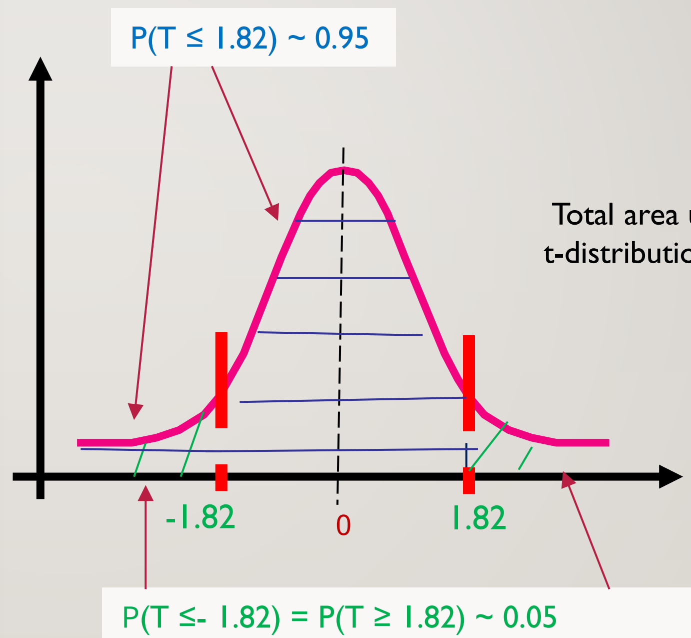
Left-Tailed Test → P-value = $P(T \leq t_0)$

ε / n	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.214
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.894
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505
				1.714	2.069	2.500	2.807	3.485
				1.711	2.064	2.492	2.797	3.467
				1.708	2.060	2.485	2.787	3.450
				1.706	2.056	2.479	2.779	3.435
				1.703	2.052	2.473	2.771	3.421
				1.701	2.048	2.467	2.763	3.408
				1.699	2.045	2.462	2.756	3.396
				1.697	2.042	2.457	2.750	3.385
				1.684	2.021	2.423	2.704	3.307
				1.676	2.009	2.403	2.678	3.261
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232
70	.254	.678	1.294	1.667	1.994	2.381	2.648	3.211

Note:
The Student's t table reports right-tail probabilities: $P(T > t)$.

P-value = $P(T \leq t_0)$ ⇔
P-value = $P(T \leq -1.82)$ ⇔ P-value = $P(T \geq 1.82)$ ⇔
P-value ~ $P(T \geq 1.895)$ ⇔
P-value ~ 0.05

The value of the test statistic is $t_0 = -1.82$



Total area under the t-distribution pdf = 1.

P-value = one green area = 0.05

EXERCISE 9.25: RIGHT-TAILED TEST

One-Sample t-Test for the Mean (Unknown Variance)

Answer:

Step 1 – Problem Setup

Right-tailed Test

- Sample size: $n = 8$
- Sample mean: $\bar{x} = 74.5$
- Sample standard deviation: $s \approx 6.23$
- Degrees of freedom: $df = 7$
- Test statistic: $t \approx -1.82$

Step 2 - Right-tailed test

- Null hypothesis: $H_0 : \mu \leq 78.5$
- Alternative hypothesis: $H_1 : \mu > 78.5$
- Significance level: $\alpha = 0.10$

Critical value: $t_{0.90;7} \approx 1.415$

RR = [1.415; +∞[

Decision rule: Reject H_0 if $t > 1.415$

p-value: P(T > -1.82) = 1 – P(T < -1.82) = 0.942

Exact value

Note:
The rejection region and the p-value will be computed in the next two slides, respectively.

Conclusion:

P-value ~ 0.942 > 0.1 → do not reject H_0

- t = -1.82 < 1.415 → **do not reject** H_0
- There is **not enough evidence** at the 10% significance level to conclude that the mean predicted score is **greater than 78.5**.

Note:
The value of the test statistic is the same for both two-tailed and one-tailed tests: $t = -1.82$.

CRITICAL VALUE $t_{1-\alpha;\ n-1}$: CALCULATION

Right-Tailed Test → RR = $[t_{1-\alpha}; +\infty[$?

ε / n	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.214
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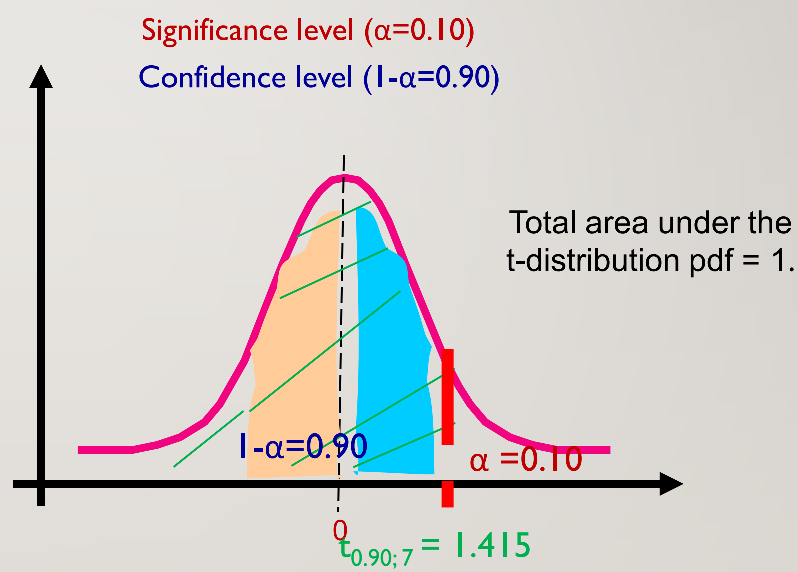
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261
				1.671	2.000	2.390	2.660	3.232
				1.667	1.994	2.381	2.648	3.211
				1.664	1.990	2.374	2.639	3.195
				1.662	1.987	2.368	2.632	3.183
				1.660	1.984	2.364	2.626	3.174
				1.658	1.980	2.358	2.617	3.160
				1.645	1.960	2.326	2.576	3.090

Note:
The Student's t table reports right-tail probabilities: P(T > t).



$\alpha = 0.10$
$1 - \alpha = 0.90$
$t_{1-\alpha;n-1} = t_{0.90;7} = 1.415$

RR = $[1.415; +\infty[$

P-VALUE FOR A STUDENT´S T-STATISTIC: CALCULATION

Right Tailed Test → P-value = $P(T \geq t_0)$

ε / n	.400	.250	.100	.050	.025	.010	.005	.001
1	.325	1,000	3.078	6.314	12.706	31.821	63.656	318.289
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.328
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.214
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.894
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852
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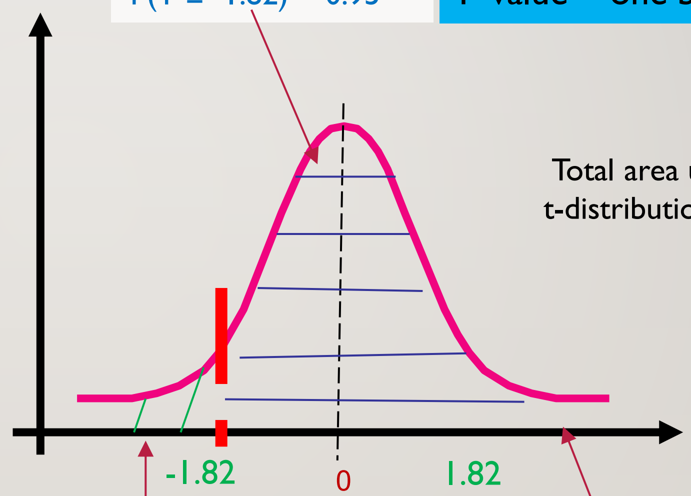
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.069	2.500	2.807	3.485
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.064	2.492	2.797	3.467
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.060	2.485	2.787	3.450
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.056	2.479	2.779	3.435
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.052	2.473	2.771	3.421
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.048	2.467	2.763	3.408
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.045	2.462	2.756	3.396
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.042	2.457	2.750	3.385
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.021	2.423	2.704	3.307
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.009	2.403	2.678	3.261
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232
70	.254	.678	1.294	1.667	1.994	2.381	2.648	3.211

Note:
The Student's t table reports right-tail probabilities: $P(T > t)$.

The value of the test statistic is $t_0 = -1.82$

$P(T \geq -1.82) \sim 0.95$

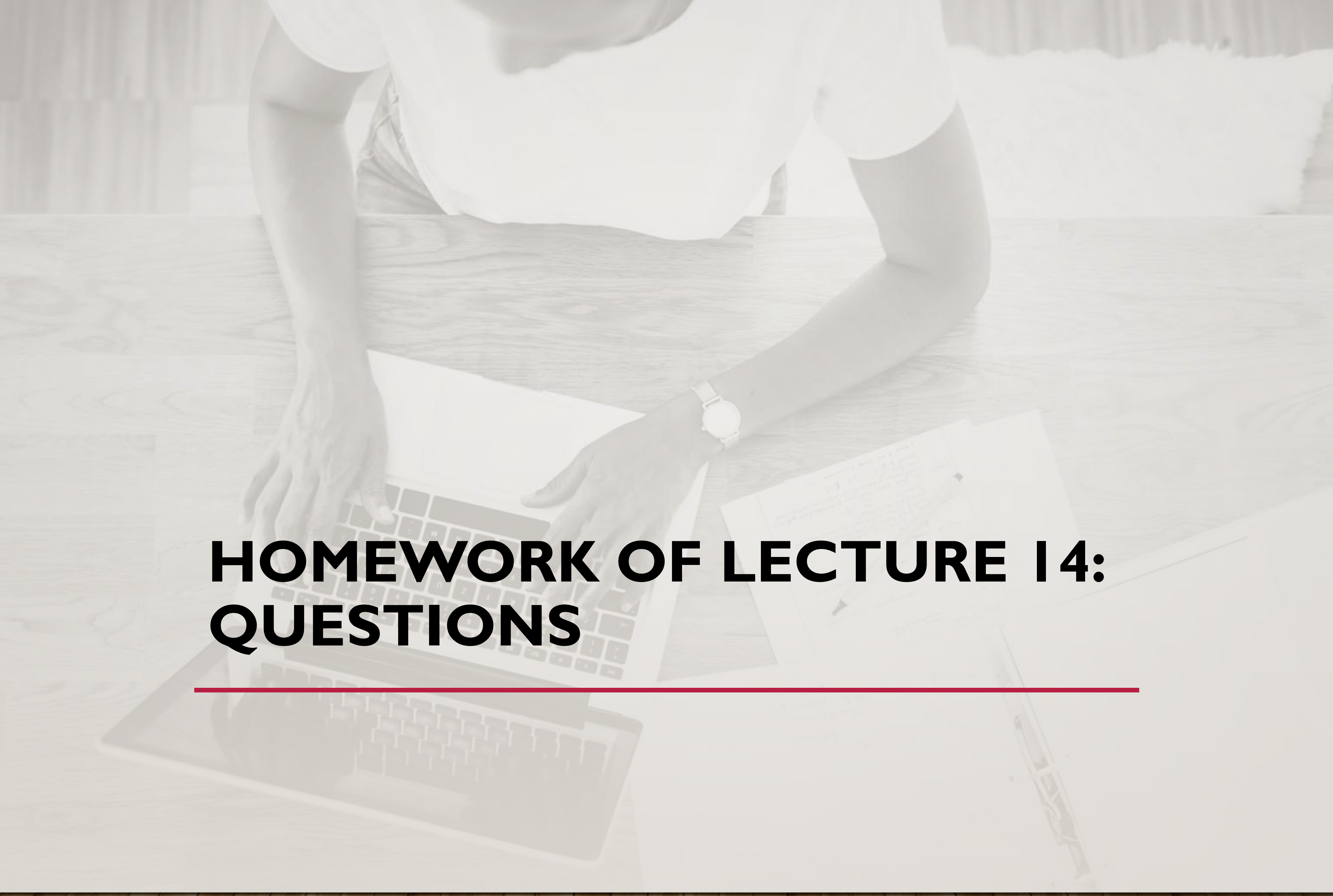
P-value = one blue area = 0.95



Total area under the t-distribution pdf = 1.

$P(T \leq -1.82) = P(T \geq 1.82) \sim 0.05$

P-value = $P(T \geq t_0)$ ⇔
P-value = $P(T \geq -1.82)$ ⇔ $P(T \leq 1.82)$
P-value ~ $P(T \leq 1.895) = 1 - P(T \geq 1.82) = 1 - 0.05 = 0.95$



HOMEWORK OF LECTURE 14: QUESTIONS

EXERCISE 9.26

9.26 An IT consultancy in Singapore that offers telephony solutions to small businesses claims that its new call-handling software will enable clients to increase successful inbound calls by an average of 75 calls per week. For a random sample of 25 small-business users of this software, the average increase in successful inbound calls was 70.2 and the sample standard deviation was 8.4 calls. Test, at the 5% level, the null hypothesis that the population mean increase is at least 75 calls. Assume a normal distribution.

Newbold et al (2013)

EXERCISE 9.26: SOLUTION

Answer:

Step 1 – Problem Setup

Left-Tailed Test

- Sample size: $n = 25$
- Sample mean: $\bar{x} = 70.2$
- Sample standard deviation: $s = 8.4$
- Null hypothesis: $H_0 : \mu \geq 75$
- Alternative hypothesis: $H_1 : \mu < 75$ (left-tailed test)
- Significance level: $\alpha = 0.05$
- Test: **one-sample t-test** (population standard deviation unknown)

Step 2 – Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{70.2 - 75}{8.4/\sqrt{25}} = \frac{-4.8}{8.4/5} = \frac{-4.8}{1.68} \approx -2.857$$

Degrees of freedom: $df = n - 1 = 24$

EXERCISE 9.26: SOLUTION

Answer:

Step 3 – Critical Value

- Left-tailed test, $\alpha = 0.05$, df = 24 → $t_{0.05,24} \approx -1.711$

Decision rule: Reject H_0 if $t < -1.711$

RR =] -∞; -1.711]

Step 4 – p-value

- Using t-distribution with 24 df:

P-value = P(T < -2.857) ~ 0.004

Step 5 – Conclusion

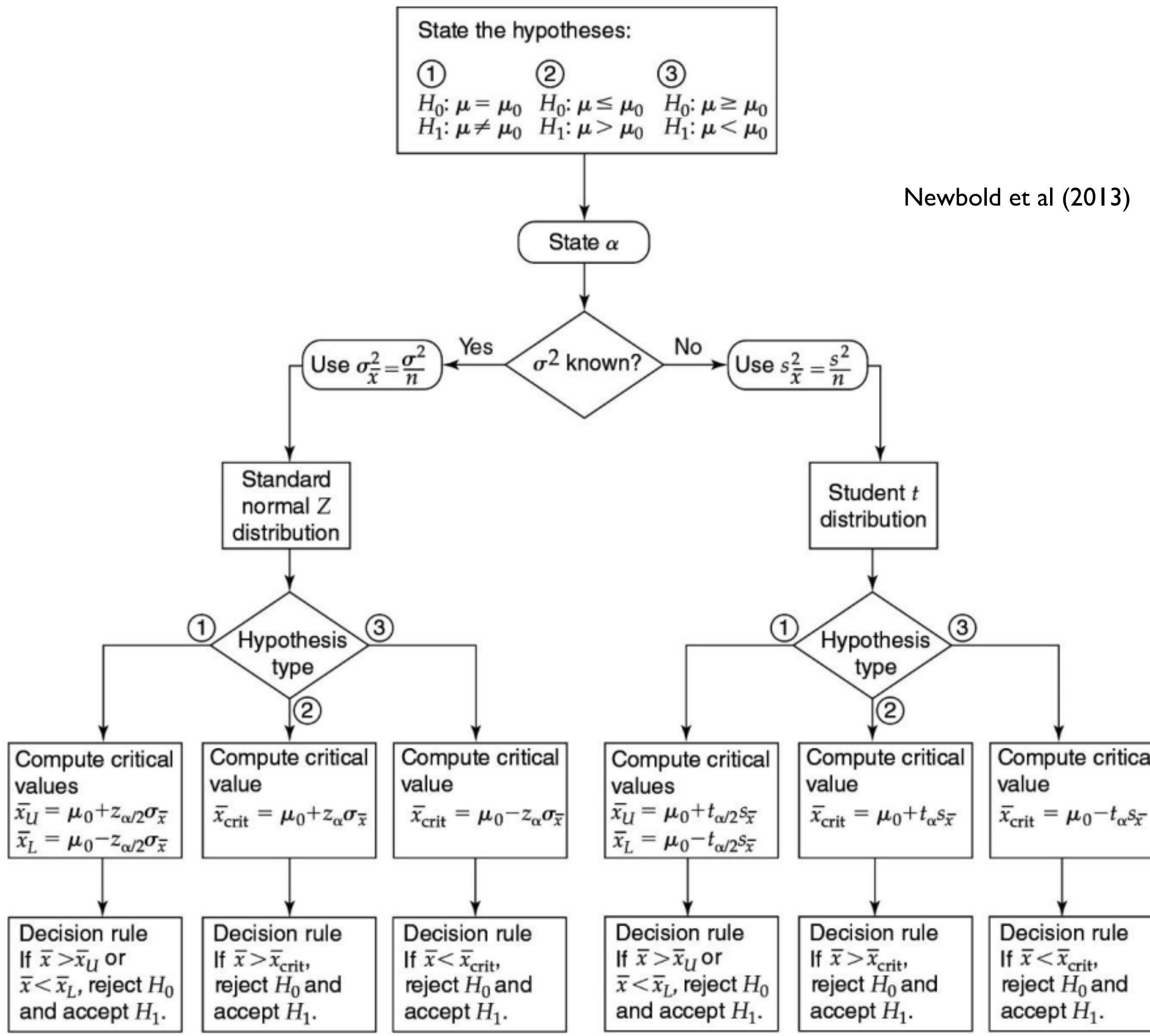
- Calculated t = -2.857 < -1.711 → **reject** H_0
- p-value ≈ 0.004 < 0.05

Interpretation: There is **sufficient evidence** at the 5% significance level to conclude that the population mean increase in successful inbound calls is **less than 75 calls per week**.

GUIDELINES FOR DECISION RULE

Figure 9.11

Guidelines for Choosing the Appropriate Decision Rule for a Population Mean



Newbold et al (2013)

LECTURE 15: TESTS OF THE POPULATION PROPORTION

TESTS OF THE POPULATION PROPORTION

- Involves categorical variables
- Two possible outcomes
  - "Success" (a certain characteristic is present)
  - "Failure" (the characteristic is not present)
- Fraction or proportion of the population in the "success" category is denoted by P
- Assume sample size is large

Newbold et al (2013)

TESTS OF THE POPULATION PROPORTION

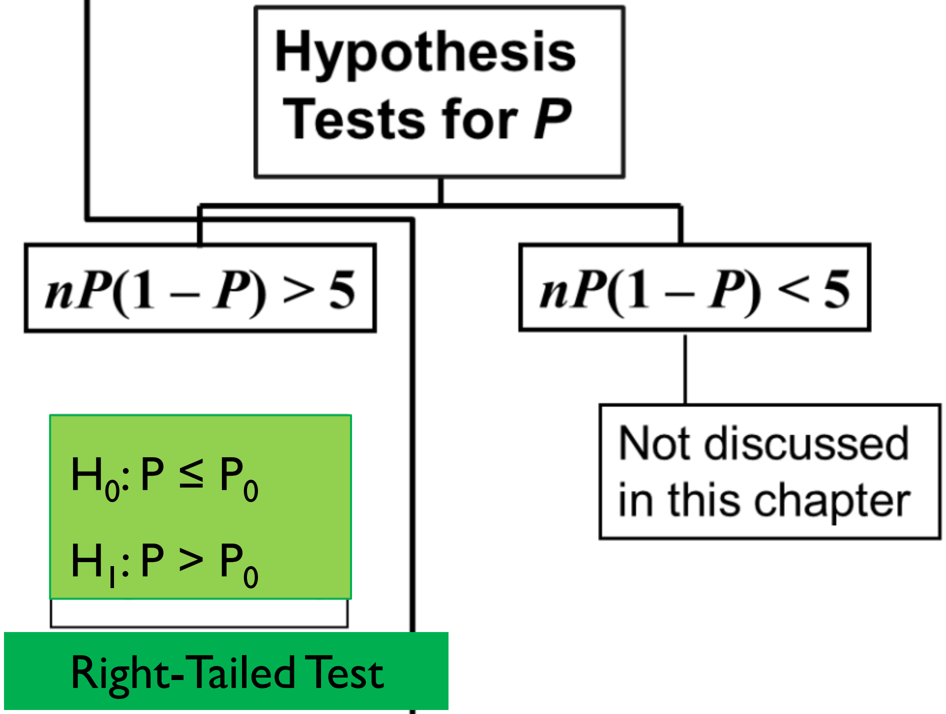
- The sample proportion in the success category is denoted by $\hat{p}$
  - $$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$
- When $nP(1-P) > 5$, $\hat{p}$ can be approximated by a normal distribution with mean and standard deviation
  - $$\mu_{\hat{p}} = P \qquad \sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

Newbold et al (2013)

HYPOTHESIS TESTS FOR PROPORTIONS: EXAMPLE

- The sampling distribution of $\hat{p}$ is approximately normal, so the test statistic is a z value:

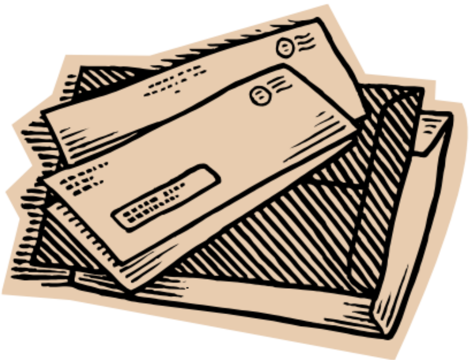
$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1 - P_0)}{n}}}$$



Newbold et al (2013)

Z TEST FOR PROPORTION: EXAMPLE

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the $\alpha = .05$ significance level.



Check:

Our approximation for *P* is

$$\hat{p} = \frac{25}{500} = .05$$

$$nP(1 - P) = (500)(.05)(.95) = 23.75 > 5$$

✓

Newbold et al (2013)

Z TEST FOR PROPORTION: SOLUTION USING THE RR

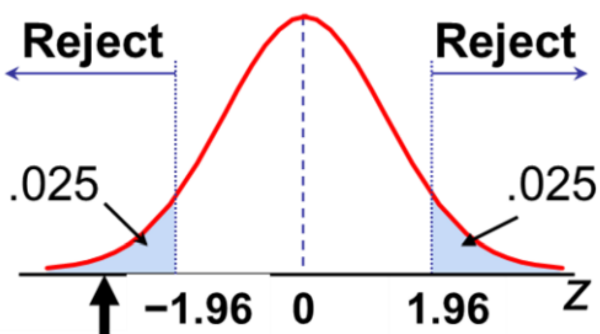
Two-Tailed Test

$$H_0 : P = .08$$
$$H_1 : P \neq .08$$

$$\alpha = .05$$

$$n = 500, \ \hat{p} = .05$$

Critical Values: ±1.96



RR =] -∞; -1.96]U[1.96; +∞[

Test Statistic:

$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$

Decision:

Reject H_0 at $\alpha = .05$

Conclusion:

There is sufficient evidence to reject the company's claim of 8% response rate.

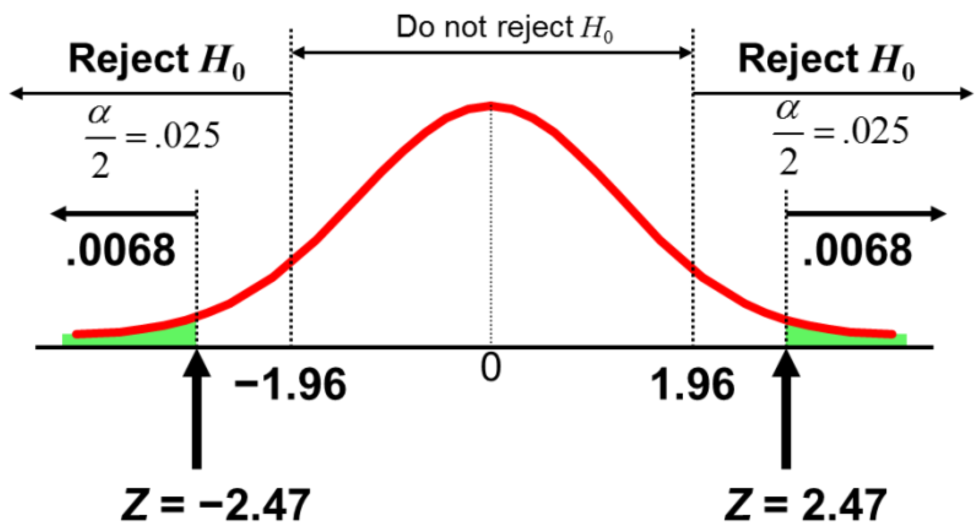
Decision using the RR: The test statistic t is in the rejection region (RR). Therefore, we reject H_0 at significance level α.

Newbold et al (2013)

Z TEST FOR PROPORTION: SOLUTION USING THE P-VALUE

Calculate the *p*-value and compare to ***α***

(For a two sided test the *p*-value is always two sided)



P-value = 2×P(T > 2.47) ~ 0.0136

$$P(Z \le -2.47) + P(Z \ge 2.47)$$
$$= 2(.0068) = 0.0136$$

Decision using the P-value:
P-value = 0.0136 < α.= 0.05.
Therefore, we reject H_0 at significance level α.

Reject H_0 **since *p*-value** $= .0136 < \alpha = .05$

Newbold et al (2013)

EXERCISE 9.30

9.30 In a random sample of 361 owners of small businesses that had gone into bankruptcy, 105 reported conducting no marketing studies prior to opening the business. Test the hypothesis that at most 25% of all members of this population conducted no marketing studies before opening their businesses. Use $\alpha = 0.05$.

Newbold et al (2013)

EXERCISE 9.30: SOLUTION

Answer:

Given

- Sample size: $n = 361$
- Number of "no marketing study": $x = 105$
- Sample proportion:

$n \times \hat{p} \times (1 - \hat{p}) = 361 \times 0.291 \times (1 - 0.291) = 74.48 > 5$

$$\hat{p} = \frac{x}{n} = \frac{105}{361} \approx 0.291$$

Note:
Since $n \times \hat{p} \times (1 - \hat{p}) > 5$, the normal approximation is valid and the one-sample proportion test can be applied.

Right-Tailed Test

- Null hypothesis: $H_0 : P \leq 0.25$
- Alternative hypothesis: $H_1 : P > 0.25$ (we are testing if **more than 25%** conducted no marketing studies)
- Significance level: $\alpha = 0.05$

EXERCISE 9.30: SOLUTION

Answer:

Step 1: Test statistic

For large samples, the **Z-test for a proportion** is:

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}}$$

Where $P_0 = 0.25$.

Standard error:

$$SE = \sqrt{\frac{0.25 \cdot 0.75}{361}} = \sqrt{\frac{0.1875}{361}} \approx \sqrt{0.000519} \approx 0.0228$$

Test statistic:

$$Z_0 = \frac{0.291 - 0.25}{0.0228} = \frac{0.041}{0.0228} \approx 1.80$$

Step 2: Critical value

Right-tailed test at $\alpha = 0.05$:

RR = [1.645; +∞[

$$Z_{0.95} = 1.645$$

Step 3: Decision

$$Z_0 = 1.80 > 1.645$$

$\Rightarrow$ **Reject** H_0 at the 5% level.

EXERCISE 9.30: SOLUTION

Answer:

Step 4: p-value

P-value = P(T > 1.80) ~ 0.0359

Since $p < 0.05$, this confirms **rejecting** H_0.

Conclusion

There is **sufficient evidence** to conclude that **more than 25%** of all small business owners who went into bankruptcy conducted **no marketing studies** before opening their business.

HOMEWORK OF LECTURE 15: QUESTIONS

EXERCISE 9.33

9.33 Of a random sample of 199 auditors, 104 indicated some measure of agreement with this statement: *Cash flow is an important indication of profitability.* Test at the 10% significance level against a two-sided alternative the null hypothesis that one-half of the members of this population would agree with this statement. Also find and interpret the *p*-value of this test.

Newbold et al (2013)

THANKS!

Questions?